

# Processes of limestone cave development

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*With plates 10 (1)–14 (5) and one figure in the text*

## Introduction

Many questions about cavern origin and subsurface water flow through limestone must ultimately be answered by a consideration of the mechanisms whereby initial fractures within limestone become enlarged into caves by the subsurface water flowing through them. Among these problems are: (1) the processes causing selective enlargement into cavern passages of only a few of the originally innumerable fractures within a limestone bed; (2) observed differences in cavern morphology not explainable by stratigraphic or structural controls; (3) the preference for cavern development directly beneath the water table; and (4) development along the shortest paths between surface sources and resurgences of groundwater.

Some theoretical approaches to groundwater flow through limestone considered the flow regime to be essentially analogous to dispersed groundwater flow within sandstone (Bretz, 1942), but White and Longyear (1962) have emphasized the necessity for considering the cavern groundwater regime as a plumbing network.

The approach to be used here is to consider the factors which affect the rate of enlargement of openings within limestone, and then use these criteria to determine the history of an enlarging cavern. Mathematically, the concern is with the instantaneous rate of enlargement of a cavern passage:

$$\frac{dD}{dt}, \quad (1)$$

where  $dD$  is a small enlargement of a cavern passage of diameter  $D$ , and  $dt$  is the time during which this enlargement occurred. In the following analyses, cavern passages will be assumed to be essentially circular in cross section. For openings of other shapes, the expressions would differ only by constants. Because of the numerous assumptions

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which must be made in order to treat mathematically the development of caverns, the mathematical expressions derived in the following sections should be regarded as being approximate solutions; these should, however, indicate the direction of passage enlargement, and the functional relationships of the factors controlling solution of limestone.

### Role of Acids Produced during Groundwater Circulation

The first stages of solution of a cavern are the most critical in its history, for the pattern of groundwater flow through the cavern and the resulting pattern of its development are determined to a great extent by the beginnings of solution. At the onset, groundwater must flow through small partings and joint openings. If an initial joint averaging 0.2 millimeters in width is assumed, with a hydraulic head of 10 meters acting over 1 kilometer, the starting groundwater velocity will be of the order of 3 centimeters per hour. If it is postulated that the principal dissolving occurs because of an original undersaturation with respect to calcite, of the groundwater entering the limestone, it is hard to imagine caves even beginning to form, for the small quantities of groundwater flowing through the joints would become almost completely saturated within the first centimeter of the joint, according to the evidence given by Weyl (1958). The first stages of joint enlargement by solution call for a different mechanism; this would likely be a local production of acid within the throughflowing groundwater or at the surfaces of the bedrock. This could either be a natural or a bacterially-assisted oxidation of organic material in the groundwater or sulfide minerals within the bedrock (Kaye, 1957).

The rate of creation of acids should be related to the concentration of oxygen and/or organic materials. In the following mathematical treatment it is assumed that the acids are produced within the throughflowing groundwater. If the acids are created at the groundwater-limestone interface, the mathematical treatment and resulting equations will be of the same type as for the groundwater flow regime which is described by equations (10) and (11).

If acids are produced within the groundwater, the supply of oxygen and oxidizable material is consumed as the groundwater flows through the joints, and the rate of creation of acid will decrease with time and distance traveled from the surface source of the groundwater. Assuming that the rate of the oxidation reactions are first-order, then:

$$\frac{dC}{dt} = -(C - C_s) \gamma, \quad (2)$$

where  $C$  is the concentration of the critical component within the groundwater (this may either be the available oxygen or the concentration of oxidizable organic material),  $C_s$  is the concentration at which no further oxidation occurs, and  $\gamma$  is a rate constant. Therefore:

$$C - C_s = (C_0 - C_s) e^{-\gamma t}, \quad (3)$$

where  $C_0$  is the concentration of the reacting component when the subsurface water first enters the limestone or passage in question. Because both the production of acid and the rate of groundwater flow through an original, or slightly enlarged joint are slow, it will be assumed that the acid will be consumed in the solution of limestone very close to its point of production. Therefore the rate of enlargement of the cavern walls will be proportional to the rate of production of acid:

$$\frac{dD}{dt} = - \left( \frac{dC}{dt} \psi \right) \cdot \left( \gamma \frac{\text{Cavern Volume}}{\text{Cavern Surface Area}} \right), \quad (4)$$

where  $\psi$  is a proportionality constant relating the rate of oxidation to the rate of production of acid, and  $\gamma$  is a proportionality constant relating the rate of acid production to the rate of increase of passage diameter. Because throughflowing groundwater will be laminar in flow during the first stages of cavern enlargement, the following relation holds:

$$\frac{h_L}{L} = \frac{32 \mu V}{\omega \bar{D}^2}, \quad (5)$$

where  $h_L$  is the hydraulic head acting across the passage,  $L$  is the length of the passage from entrance to exit of groundwater,  $\bar{D}$  is the effective diameter of the cavern passage,  $V$  is the average velocity of the throughflowing groundwater, and  $\mu$  and  $\omega$  are constants. But since velocity is distance traveled over a period of time, equations (2)-(5) may be combined to obtain:

$$\frac{dD}{dt} = \psi \gamma (C_0 - C_s) \frac{D}{x} e^{-\frac{32 \gamma \mu x L}{\omega h_L \bar{D}^2}}, \quad (6)$$

where  $x$  is the position within the passage measured from the origin (i.e.,  $0 \leq x \leq L$ ). This formula does not yield a simple integral, but the main features of the pattern of enlargement may be seen from the equation. Three cases are possible.

1.  $(32\gamma\mu \times L) \gg (\omega h_L D)$ . In this case the exponential term is essentially zero, and no significant passage enlargement will occur. This will be true for very small initial joint diameters, and for joints with little hydraulic gradient along them. If no oxidation reactions occur (when the oxygen or oxidizable material is exhausted, or when the rate of oxidation is very slow), or if no acids are produced by the oxidation reaction, then there will also be no solution of limestone by this mechanism.

2.  $(32\gamma\mu \times L) \approx (h_L \omega \bar{D}^2)$ . In this case joints with greater hydraulic gradient along them and larger initial diameter will be more greatly enlarged than those joints with lesser hydraulic gradients and smaller widths. The rate of joint enlargement will decrease along the path of groundwater flow (as  $x$  increases.)

3.  $(32\gamma\mu \times L) \ll (\omega h_L \bar{D}^2)$ . In cases where this situation holds, no significant reduction of local acid production will occur along the path of flow because of high hydraulic gradient and/or large joint diameter. Thus the exponential term will be essentially unity, and the rate of increase of joint diameter will be proportional to the diameter.

Initial stages of cavern development will occur only along joints or joint complexes describable by cases 2 or 3. Thus many initial fractures in limestone will have insufficient hydraulic gradient along them or are of too small an original diameter to be enlarged into cavern passages. Therefore there is an initial favoring for enlargement into cavern passages of joint systems of short connection between a source of groundwater and its resurgence to the surface, of originally larger fracture openings, and of passages oriented parallel to the hydraulic gradient. In the early stages of cavern formation the hydraulic head,  $h_L$ , will be essentially constant, and the water table will lie close to the ground surface, because not all surface drainage can be diverted underground. Therefore the rate of joint enlargement will increase with time as the lower term of the exponent becomes larger.

### Role of Original Undersaturation of Groundwater

As the embryonic cavern passages grow in size and the through-flowing discharge increases, eventually the dominant dissolving process will be by virtue of the original undersaturation of the groundwater with respect to calcite, rather than by locally produced acids. Limestone is soluble in groundwater both because of the solubility of calcite in pure water and because of carbonic acid within groundwater which is contributed from the atmosphere and from soil sources. Limestone placed within an acidic solution reaches equilibrium more rapidly than

sulfides or organic material are oxidized by dissolved oxygen. Thus, when groundwater flow becomes rapid enough such that the groundwater flowing through the enlarging joints is not completely saturated with respect to calcite, there should be a rapid increase in rate of dissolution of limestone, and only those joints which have been enlarged sufficiently to become affected by the new process should receive most future cavern development. Therefore, through the action of this "threshold," only a selected few of the numerous original limestone fractures will become enlarged into cavern passages; these will be those fractures or fracture complexes which have acting across them originally a high hydraulic gradient, and/or are of initially larger diameter. Within local areas of limestone the original fracture diameters will be of greatest importance, because nearby parallel joints will have nearly the same hydraulic gradients. But on a larger scale, in the absence of unusual structural controls, the distribution of hydraulic gradient will be the predominant factor.

Where solution of limestone occurs because of the initial undersaturation of the groundwater with respect to calcite, the rate of solution will be a function of the average concentration of dissolved calcite relative to the saturation concentration of calcite. Although the rate of solution is in actuality a complex function of concentration gradients (Weyl, 1958), a first-order reaction will be assumed here. Thus the following describes the change of composition of the groundwater with time:

$$\frac{dC}{dt} = -\frac{4\delta(C_s - C)}{D} \quad (7)$$

where  $\delta$  is a rate constant,  $D$  is the diameter of the passage through which the groundwater flows, and  $C_s - C$  is the average departure from saturation of the groundwater. Therefore:

$$\bar{C} - C_s = (C_0 - C_s) e^{-\frac{4\delta t}{D}} \quad (8)$$

where  $\bar{D}$  is the effective diameter of the passage through which the groundwater passes in a time  $t$ . The rate of increase of passage diameter will be proportional to the rate of solution of calcite multiplied by the ratio of passage area to passage circumference:

$$\frac{dD}{dt} = -\frac{\pi D}{4} \cdot \frac{dC}{dt} \quad (9)$$

where  $v$  is a proportionality constant. Substituting within equations (5) and (7)-(9), and assuming that the effective diameter for groundwater solution in equation (8) is equal to the effective diameter for groundwater motion in equation (5), the following relationship is found:

$$\frac{dD}{dt} = v \delta (C_0 - C_s) e^{-\frac{128 \delta D \times L}{D^3 h_1 \omega}} \quad (10)$$

This equation is similar in form to equation (6), and the type of cavern development will again depend upon the value of the exponent. If the hydraulic gradient and passage diameter are small, little enlargement of the passages will occur by this groundwater regime, and local acid production along the passage would of necessity account for any solution. At least in the early stages of this groundwater regime, the hydraulic head will be constant, and the rate of increase of passage diameter will rise until the exponential term is essentially equal to one, where it will become constant.

As enlargement of the primitive cavern passages continues, the discharge will increase until all available surface drainage has been diverted underground. When this occurs the flow through the passage network will become essentially constant, but the hydraulic head will decrease as the passages enlarge. In some groundwater situations, limited available discharge may be felt when the groundwater regime is describable by equation (10). Eliminating the variable of hydraulic head from this equation and substituting that of discharge gives the following relationship:

$$\frac{dD}{dt} = v \delta (C_0 - C_s) e^{-\frac{4 \pi \delta \times D}{Q}} \quad (11)$$

where  $Q$  is the discharge. If discharge through the cavern passage is sufficient, the exponential term will be essentially equal to unity, and the rate of cavern enlargement will be essentially constant. Where available discharge must be divided among the various passages leading from source to resurgence of groundwater, the share of discharge carried by each cavern passage will be an inverse function of the length of the alternative paths and a direct function of the diameter of the cavern passages. Thus only certain passages which, during earlier stages of solutional activity were enlarged the greatest amount, will experience the maximum amount of solution under constant discharge conditions.

### Role of Turbulent Flow

If available discharge is sufficient, a transition to turbulent flow in the largest of the primitive passages may occur. Because in turbulent flow the throughflowing groundwater is mixed, the rate of solution is several times that occurring with laminar flow, and only those passages favored by earlier solutional regimes so that turbulence may begin in them will receive significant further enlargement (White and Longyear, 1962). Thus the laminar-turbulent threshold of flow further reduces the number of significantly enlarged passages relative to the number of passages which had been enlarged by earlier solutional regimes.

Kaye (1957) has shown in several experiments with strong acids that the rate of solution of limestone in turbulent flow is a function of the rate of motion of the solvent. Although Kaye's experiments are not numerically applicable to groundwater kinetics, a theoretical approach may be formulated.

In turbulent flow the velocity distribution is similar to figure 1. Near the wall of the passage is a thin layer of laminar flow whose thickness is  $\Delta$ . It may be assumed that turbulence brings all of the throughflowing groundwater to the same concentration inside this thin laminar

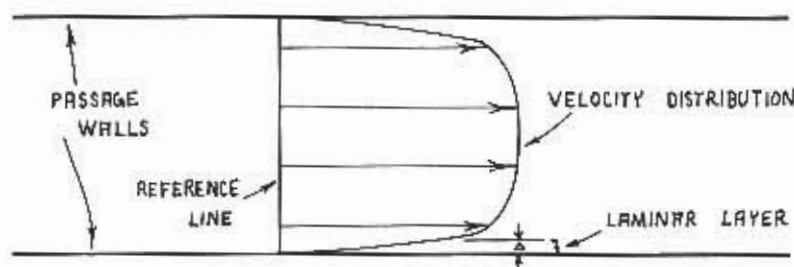


Fig. 1

zone, hence there is a concentration gradient from an unsaturated concentration  $C$  of dissolved limestone within the turbulent zone to saturation  $C_s$  at the limestone wall. Assuming that migration of dissolved limestone to the turbulent zone will be by diffusion across the laminar zone, the following holds:

$$\frac{dD}{dt} = \beta \frac{dm}{dt} / \text{unit area}, \quad (12)$$

where  $\frac{dm}{dt}$  is the rate of mass diffusion across the laminar layer, and  $\beta$  is a proportionality constant. In such a case the following is true, assuming a uniform concentration gradient across the laminar zone:

$$\frac{dD}{dt} = \frac{(C - C_s) \beta z}{\Delta}, \quad (13)$$

where  $z$  is a diffusion constant. The following equations describe turbulent flow in a smooth tube of constant diameter:

$$N_R = VD^{\mu/\mu}, \quad (14)$$

$$f = \frac{3164}{N_R^{1/4}}, \quad (15)$$

$$h_L = f \frac{LV^2}{D_0 g}, \quad (16)$$

$$\Delta = \frac{5 \mu \sqrt{g}}{V \beta \sqrt{f}}, \quad (17)$$

where  $N_R$  is the Reynold's Number, and  $f$  is a friction factor.

Because of the rapidity of groundwater flow in the turbulent regime, it may be assumed that the groundwater does not approach saturation during its trip through the cavern system. If so, the term  $(C - C_s)$  will be essentially a constant. For the purpose of yielding a simple solution, the passage diameter will be assumed to be uniform in the direction of flow. This latter assumption may be made because position along the cavern passage does not enter in to the final expression. Therefore, equations (13)–(17) may be solved to eliminate  $N_R$ ,  $f$ ,  $\Delta$ , and  $V$  to obtain, assuming  $h_L$  to be a constant:

$$D^{1/2} = D_0^{1/2} + \frac{z \beta (C - C_s)}{20} \left( \frac{g h_L}{L} \right)^{1/2} \left( \frac{z}{\mu} \right)^{3/4} t, \quad (18)$$

where  $z$ ,  $\beta$ ,  $\mu$ , and  $g$  are constants, and  $D_0$  is the original cavern diameter at time  $t = 0$ . As long as not all available discharge is diverted underground ( $h_L$  is a constant) the rate of increase of passage diameter will increase with time, and passages with a greater hydraulic gradient acting across them (greater hydraulic head or shorter length) will be enlarged the greatest amount, because such passages will have a

greater velocity of throughflowing groundwater. This equation describes only passages with smooth walls. Cavern passages with rough walls will have more complex velocity functions.

Eventually, for caverns forming within the turbulent groundwater flow regime, all available drainage must be diverted underground, and the discharge  $Q$  rather than the hydraulic head  $h_1$ , will be essentially constant. The equivalent expression for cavern development will be:

$$D^{2.88} = D_0^{2.88} + \frac{8}{2.3} \times \beta (G - C_s) \left( \frac{\rho}{\mu} \right)^{1/2} \left( \frac{1}{\pi} \right)^{1/2} \frac{\sqrt{3164}}{51.8} t, \quad (19)$$

where  $\bar{Q}$  is an effective discharge taking into account the seasonalities of discharge. In cavern situations described by this case the rate of cavern enlargement decreases as the cavern passage enlarges, but should never fall below the constant enlargement rate for laminar flow given in equation (11). Where drainage underground is divided among available passages, those passages with greatest hydraulic gradient and largest initial passage diameter will receive the greatest share of the discharge, and therefore the greatest enlargement under the above groundwater regime.

### Discussion

On plate 10 (1) are generalized the hydrochemical features of groundwater flow. The ordinates are the effective width, in centimeters, of the passage, and the hydraulic gradient, a dimensionless ratio of the hydraulic head acting across the passage to the length of the passage. Both are plotted logarithmically. The laminar-turbulent transition is assumed to lie at a Reynold's Number of about 2000. The other boundary line is based upon Weyl's (1958) data on penetration distance to 90% saturation. For the zone to the right of the line (greater velocity and diameter), the penetration distance is greater than 3 meters, and less for the area to the left. At constant hydraulic head the distance is proportional to the fourth power of the diameter, and is therefore a critical boundary of the zone where solution by virtue of the original undersaturation of groundwater is dominant. The line which marks the limit of significant solution is approximate, and represents the lowest combination of hydraulic gradient and passage diameter where solution by locally produced acids will be important. Lines of equal velocity are also shown. For computation of velocities in the turbulent zone, a rough-walled tube was assumed.

Plate 11 (2) shows possible paths of development of a major cavern passage. Until the limited discharge is felt, cavern development will be at constant head. Initial enlargement of cavern passages will be by acids produced locally along the passage (equation 6), and then the regime will progress to solution by virtue of the initial undersaturation of the groundwater when it enters the limestone (equation 10), and, if sufficient discharge is available, turbulent flow will be established (equation 18). During enlargement under constant head, the rate of cavern enlargement constantly increases, with order-of-magnitude discontinuities at the onset of the successive processes. Constant-discharge conditions may begin while the flow is laminar (case A; equation 10) or turbulent (case C; equation 19). An intermediate case is possible (case B). When constant discharge becomes established, the hydraulic gradient, velocity of throughflowing groundwater, and rate of cavern enlargement are decreasing functions of time. The solutional history of a simple cavern which experienced constant discharge while groundwater flow was laminar is shown in plate 12 (3) A and B illustrates a cavern in which groundwater flow became turbulent.

Case D of plate 11 (2) represents a special case where effective passage diameter remains constant after reaching a critical size. Two groundwater situations may promote a constant-diameter cavern passage. The first of these is where sediment enters a cavern at a high rate, and tends to fill in the cavern passage. The velocity of the groundwater will be able to keep only the topmost level of the cavern free from sediment, and a constant-rate upward solutional enlargement of the cavern passage will be accompanied by sedimentation. The second case is the similar, but opposite-acting case where groundwater flows as a free-surface, high gradient subterranean stream. Such a free-surface stream can develop only under constant-discharge conditions, and, given homogeneous limestone, the wetted perimeter of the cavern passage will remain constant as it becomes enlarged by downward solutional erosion while maintaining a constant width.

The development of two simple types of cavern systems will now be considered. The first of these is the ordinary topographic-geologic situation which eventually leads to cavern growth directly beneath a nearly flat water table, and the other is cavern development under artesian flow.

### Water-Table Caverns

In plate 13 (4) is shown a physiographic situation which promotes cavern development. For this series of diagrams it is assumed that this situation has been obtained without previous solution of lime-

stone, and that cavern development occurs without accompanying change of surface topography. A small stream has also been assumed to be present on the upland surface; this stream will be perched with respect to the "intrenched" stream to the right in plate 13 (4). Before significant solutional enlargement of joints occurs, the groundwater level is almost directly below the surface in all but arid climates. In the initial stages of groundwater circulation, the groundwater flow through the joint openings will be essentially similar to groundwater flow in porous rocks, following a deep arcuate between the upland surface and the lower stream level (plate 13 [4] A). In the initial stages of solution by locally produced acids, the greatest amount of solutional activity will presumably be near the edge of the upland where the hydraulic gradient is high, and the rate of passage solution may be described by equation (6).

As solution increases the joint widths, the limited discharge available for underground diversion will first be felt near the edge of the upland because of the greater rate of solution there, and the level of the water-table will drop in that area (dotted line in plate 13 [4] A). In plate 13 [4] A are assumed two primitive flow lines ABO and CDO superimposed upon the later water-table. Across the line BD there is a strong hydraulic gradient, and more solution occurs along flow lines to the right where the initial hydraulic gradient was greatest: hence flow lines become shallower where the water table slopes downward near the edge of the upland (plate 13 [4] B). The upland stream continues to contribute groundwater from a constant hydraulic head for a long time after constant discharge has become established over most of the upland, and drainage from the upland stream will contribute an increasing proportion of the underground drainage. Eventually solutional enlargement of passages may be sufficient that beneath most of the upland area the hydraulic head will be negligible compared to that beneath isolated entrances of underground drainage, and the water table will essentially become graded to these larger groundwater entrances (plate 13 [4] C). Almost all groundwater movement will occur directly below the nearly flat water-table. Adding to the effect of the shallowing of the zone of groundwater movement will be the restriction of major solution to only a few cavern passages because of the selective effects of the transition to predominant solution by originally undersaturated water and to turbulent flow. Note the similarity of this concept of cavern development to that of Rhoades and Sinacori (1941). Moore and Nicholas (1963) propose another mechanism which also tends to increase the relative solution directly beneath the water table: soil water containing carbonic acid

moves downward in the zone above the water table too fast to utilize more than a small part of its dissolving capacity, but at the top of the water table it may remain in contact with the limestone long enough to become fully saturated before moving onward.

Near the entrances of surface drainage to underground flow, the movement of groundwater has a strong vertical component, and is likely to consist of turbulent flow in steep, free-surface underground streams and even low waterfalls and dome pits.

In caverns in the United States, an episode of sediment aggradation within caverns, followed by varying degrees of subsequent removal of the sediments, is commonly found. It is proposed here the introduction of sediments is often contemporaneous with the later stages of solutional activity. The influx of sediments tends to fill in lower solutional channels and floor irregularities, and there is a tendency to dissolve upward which is limited by the level of the water table (see discussion of plate 11 [2], case D). In caverns with a steep water-table gradient, or in caverns where the outlet level has dropped, capture of discharge by lower passages may occur, and upper levels may be abandoned except in time of flood. This flood-stage flow tends to wash out sediments deposited earlier.

The tendency toward creation of a nearly level water-table with maximum solution directly beneath should be common to all cavern-forming situations free from unusual structural, stratigraphic, or topographic controls.

### Caves Formed by Artesian Flow

Another significant pattern of flow occurs in artesian situations, where groundwater is forced to follow a deep path between its entrance and exit by stratigraphic and structural constraints. In the idealized case (plate 14 [5] A), groundwater is assumed to rise to the surface along the steep limb of the asymmetric anticline and exit to the surface from a single point. An artesian groundwater gradient will become established when the top of the anticline in limestone is intersected by surface drainage. At first, before significant solution occurs, the water table in the limestone will coincide with the ground surface in the groundwater collection area (plate 14 [5] B). The paths of initial groundwater flow will be essentially as in this figure, although the rate of solution will be greatest along the shortest connection between the water table and the water outlet (along the dip). Such a flow pattern will be continued as long as all drainage has not been diverted underground. Constant discharge will occur earliest along the shortest path

between entrance and resurgence of groundwater. Therefore a draw-down of the water-table will occur in this area. This will cause a convergence of drainage toward the shortest drainage channels along the dip (plate 14 [5] C). Assuming a uniform pressure gradient along the two embryonic passages ABO and CDO, it can be seen that there will be a pressure gradient across connecting passage BD, and CDO will tend to lose drainage to ABO. Because this is an escalating process, a strong horizontal flow component will be established just below the water-table, with corresponding development of water-table passages (plate 14 [5] D). Artesian caverns would be more maze-like than water-table caverns because of the difference in the scale of flow between the two groundwater flow patterns. In artesian caverns, passages over a wider zone will have an equivalent hydraulic gradient than will be the case in shorter water-table caverns.

#### ABSTRACT

Three processes successively predominate in enlarging original fractures within limestone into cavern passages: (1) early dissolving by acid produced by oxidizing reactions within the groundwater as it flows through the limestone; (2) dissolving caused by the initial undersaturation with respect to calcite of the groundwater when it enters the limestone; and (3) increased dissolving which occurs at the transition from laminar to turbulent groundwater flow.

Only those original fractures in limestone which are widest and which have a high hydraulic gradient acting across them will be enlarged into cavern passages. Until all available surface drainage has been diverted underground, cavern development takes place under a constant hydraulic head, and the rate of limestone solution increases with time. After all available surface drainage has been diverted underground, the discharge through the cave, rather than the hydraulic head, remains constant, and the rate of limestone solution decreases toward a constant value.

These principles apply to caverns formed both by water-table flow and by artesian flow.

#### ZUSAMMENFASSUNG

Drei Vorgänge werden unrissen, die nacheinander als die vorherrschende Kraft bei der Auflösung natürlicher Risse im Kalkstein zu Höhlendurchgängen sich auswirken: 1. eine frühe Periode der Säurebildung durch Oxydationsreaktion in dem durch den Kalkstein fließenden Grundwasser, 2. Lösung auf Grund der ursprünglichen Kalkspatuntersättigung des Grundwassers, wenn es mit dem Kalkstein in Berührung kommt, und 3. der Übergang zu einem heftig bewegten Abfluß des Grundwassers.

Nur diejenigen natürlichen Risse im Kalkstein, die die größte Weite haben und unter hohem hydraulischem Druck stehen, werden zu Höhlendurchgängen ausgeweitet. Bis die gesamte zur Verfügung stehende Oberflächenentwässerung zum Untergrund hin abgeleitet worden ist, findet der Ausbau der Höhle unter gleichbleibendem hydraulischem Druck statt, und

das Maß der Kalksteinauflösung nimmt mit der Zeit zu; nachdem die gesamte verfügbare Oberflächenentwässerung zum Untergrund hin abgeleitet worden ist, wird anstatt des hydraulischen Drucks der Abfluß durch die Höhle konstant bleiben, und das Maß der Auflösung des Kalksteins wird sich auf einen gleichbleibenden Wert vermindern. Die Geschichte vereinfachter Grundwasser- und artesischer Höhlen ist unrisen.

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## EXPLANATION OF PLATES 10 (1)-14 (5)

### PLATE 10 (1)

Hydrochemical groundwater regimes in terms of passage width, hydraulic gradient, and groundwater velocity.

### PLATE 11 (2)

Paths of cavern-passage enlargement: (A) constant discharge experienced during laminar groundwater flow; (B) intermediate case showing reversion to laminar flow; (C) constant discharge experienced during turbulent groundwater flow; (D) constant passage diameter.

### PLATE 12 (3)

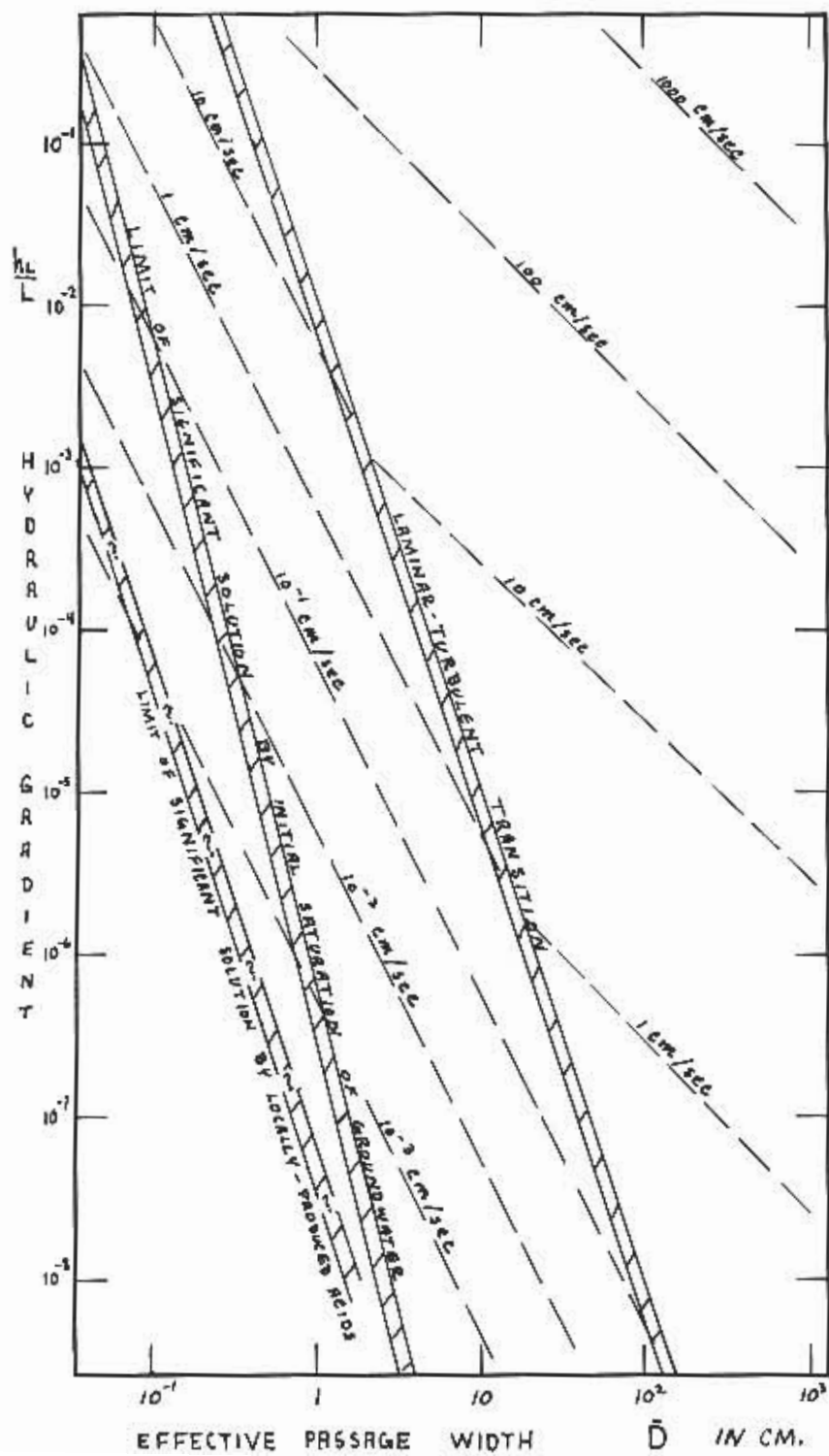
(A) theoretical path of enlargement of a cavern passage which does not experience turbulent flow; (B) theoretical path of enlargement of a cavern passage in which a transition to turbulent flow occurs.

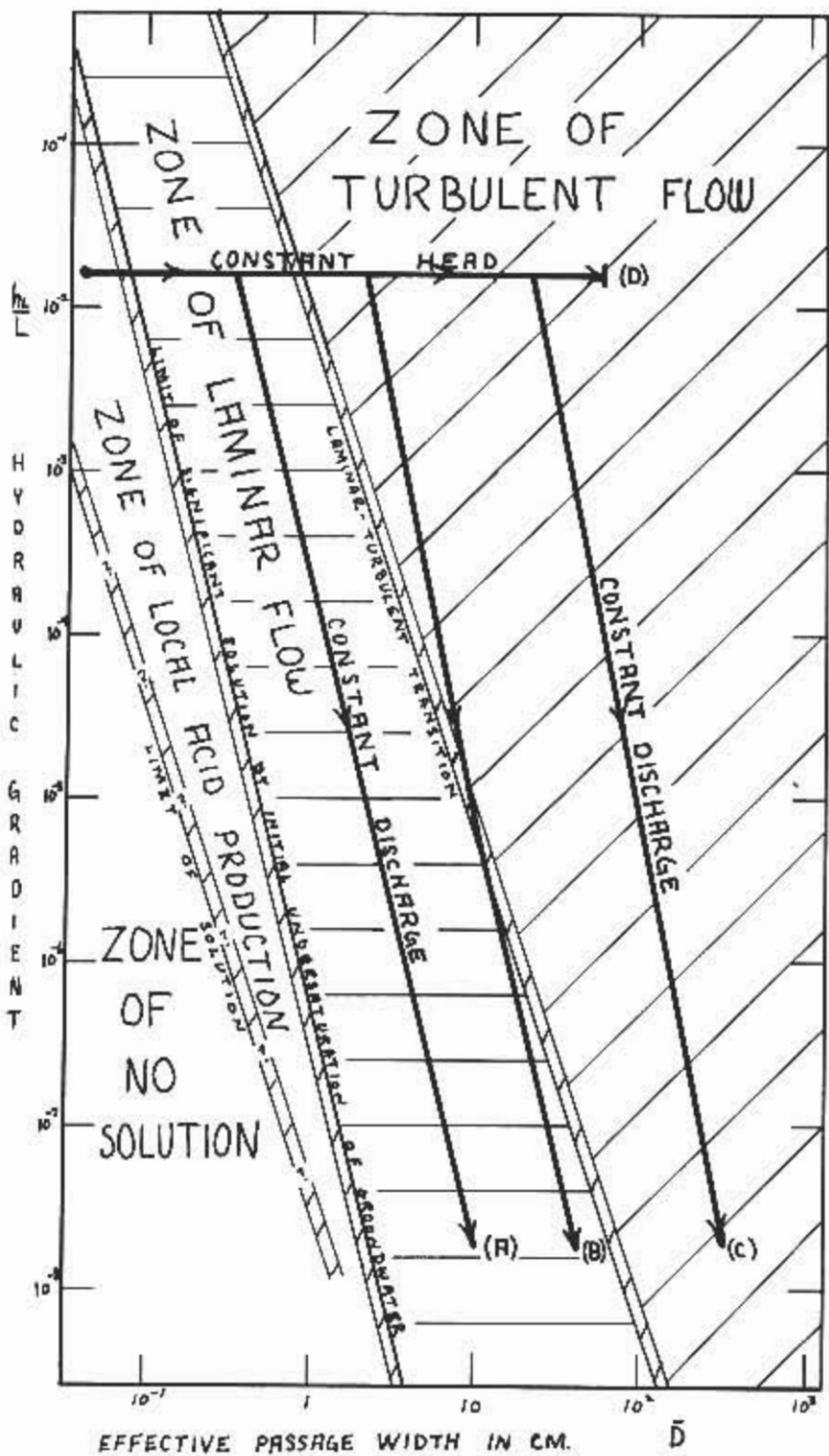
### PLATE 13 (4)

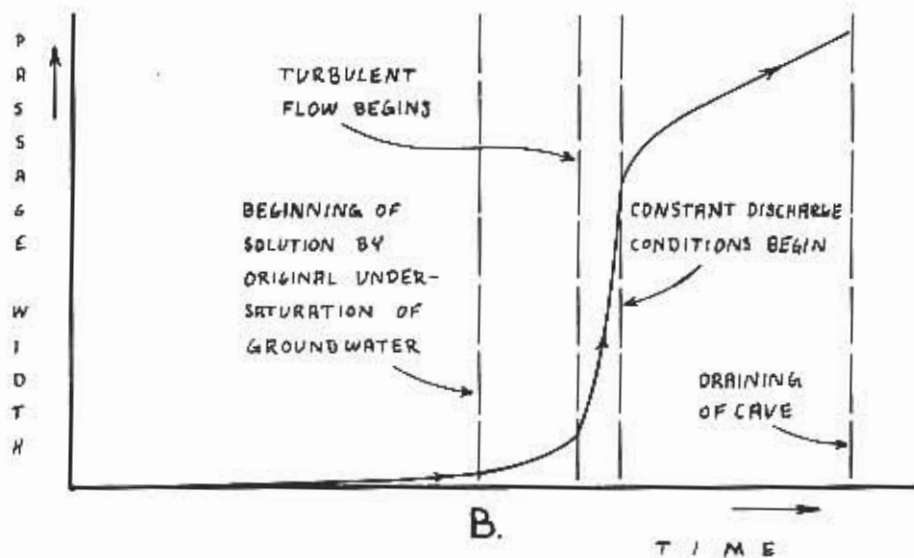
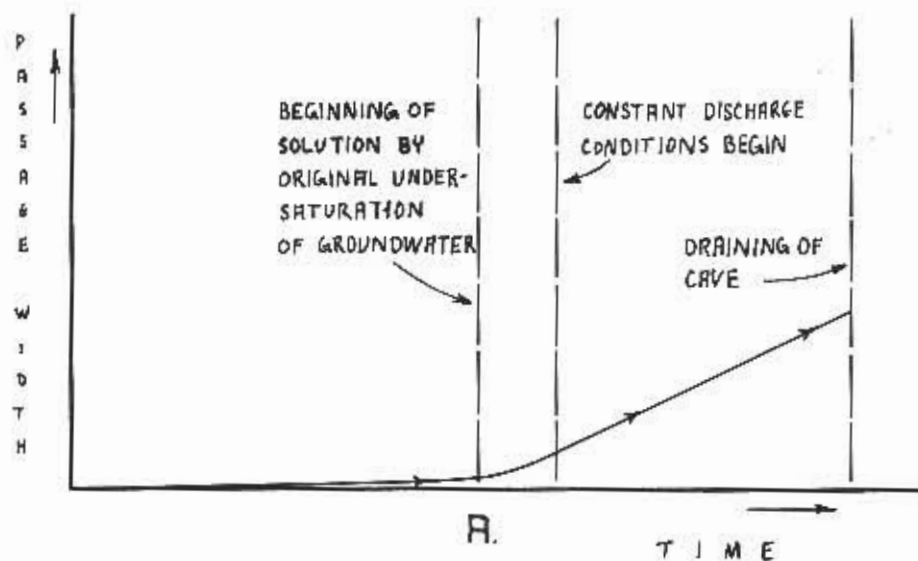
A cross-section through homogeneous limestone, showing successive stages of the development of a water-table cavern.

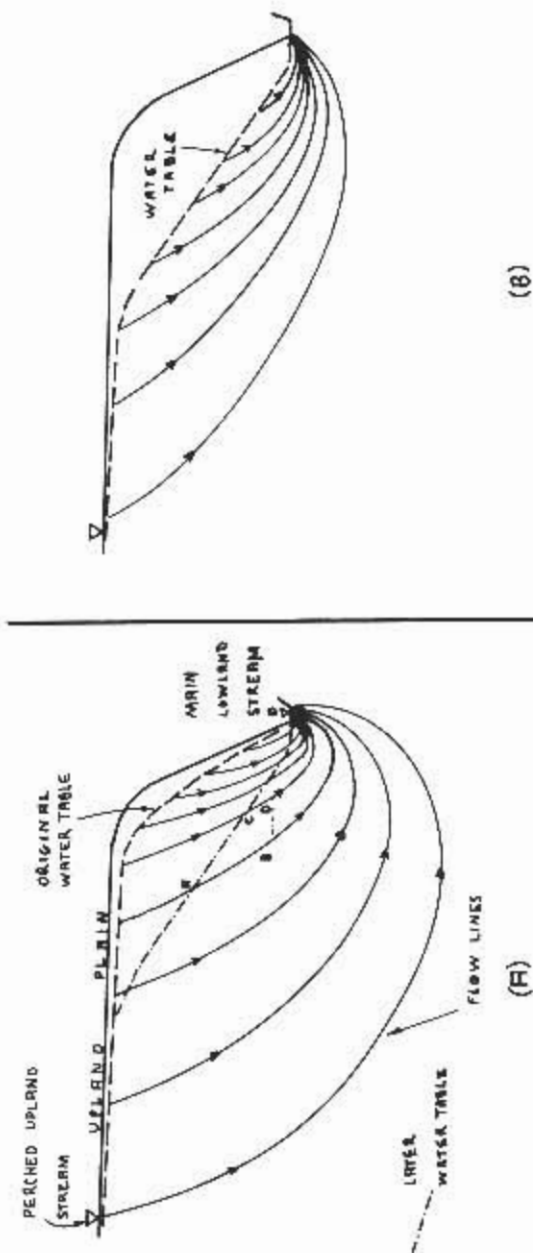
### PLATE 14 (5)

Development of an artesian cavern: (A) cross-section showing structural and topographic conditions; (B)-(D) plan views showing successive stages of the development of an artesian cavern.

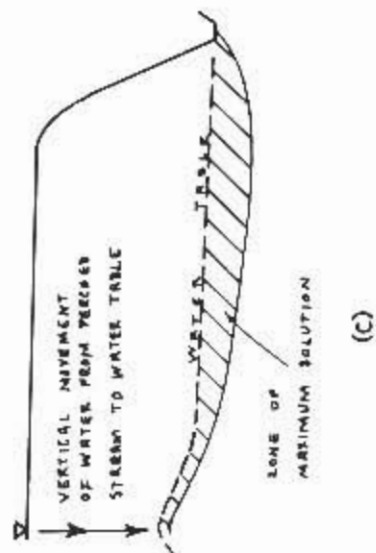




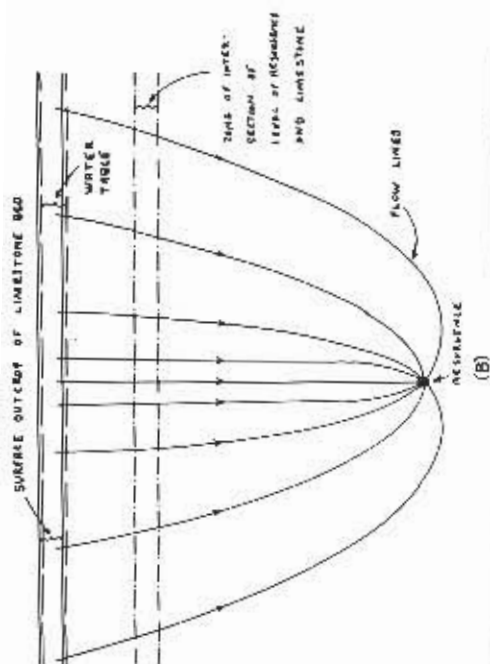




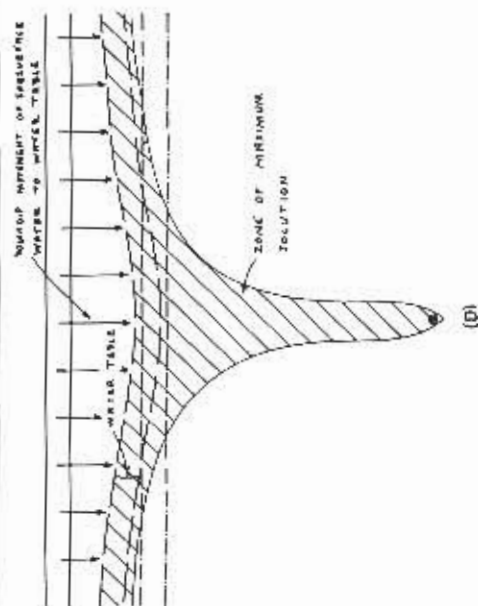
(B)



(C)



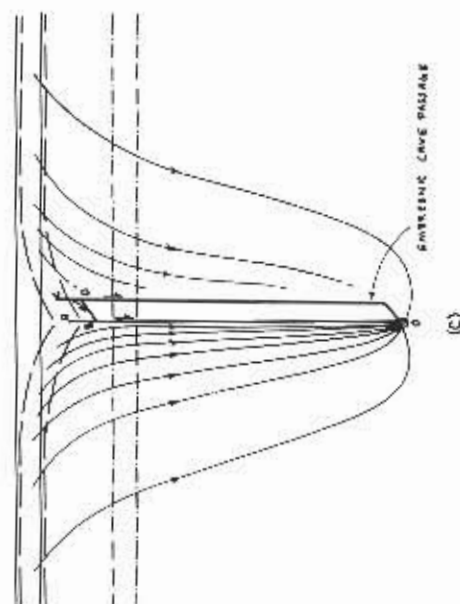
(B)



(D)



(F)



(C)